An Improved XFEM for Field Analysis of Multilayer HTS Tapes with Multiple Nearby Geometrical Interfaces

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This paper proposes an improved extended finite element method (XFEM) for modeling high temperature superconducting (HTS) tapes with multiple nearby geometrical interfaces. In regions near these interfaces, the magnetic vector potential approximation is enriched by incorporating multiple derivative discontinuous fields based on the partition of unity method such that the interfaces can be represented independent of the mesh. The support of a node or an element can be cut by several interfaces. This method results in the high accuracy in the approximation field and the derivative field. Numerical examples applied to the multilayer HTS tapes in 2D eddy current field involving level set based parts, error analysis and electromagnetic field computations are provided to demonstrate the utility of the proposed approach.

***Index Terms*— Eddy current, high order enrichment function, nearby geometrical interfaces, XFEM.**

I. Introduction

FOR MANY electromagnetic devices, the complex components in structure contain large number of nearby geometrical interfaces. The basic structures of some examples are shown in Fig. 1, where (a) and (b) are the laminated iron cores in a transformer and a motor, (c) is the superconducting layers in a high temperature superconducting (HTS) cable, and (d) the magnetic particles in a magneto-rheological fluid.

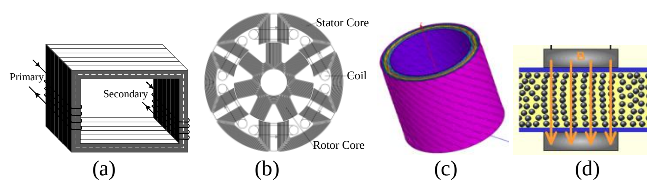


Fig. 1. Examples of electrical devices with multiple nearby interfaces: (a) laminated iron core in a transformer, (b) laminated iron core in a motor, (c) superconducting layer in a HTS cable, and (d) magnetic particles in a magneto-rheological fluid.

Meshless method [1] is an answer to the meshing issue, as the connectivity is obtained by means of domains of influence that are not mesh based. However, the computational cost of this approach is still higher than that of conventional finite element method (CFEM), and the parameters involved in the formulation are not always easy to select a priori.

The extended finite element method (XFEM) [2-6] with a single level set function to construct the enrichment function can be very effective for the case when the elements across the boundary of two different media contain only one interface. However, when the elements contain more interfaces due to the small sizes of media, such as the laminated cores in transformers and motors and the superconducting layers in HTS cables, it would encounter many numerical difficulties. To overcome these limitations, an improved XFEM is proposed.

II. Improved XFEM with Multiple High Order Enrichment Functions and Low Order Meshing Element

Fig. 2 shows a portion of a mesh with quadrilateral elements, where Γ is the interface, which does not necessarily coincide with the mesh, and n_i are the enrichment nodes (circles) whose supports ω_i are cut by the interface. The elements cut by the interface are known as the enrichment elements, and the elements with enrichment nodes which are not cut by the interface are known as the blending elements. ω_i=supp(n_i) is the support of node n_i, which consists of the union of all elements connected to node n_i.

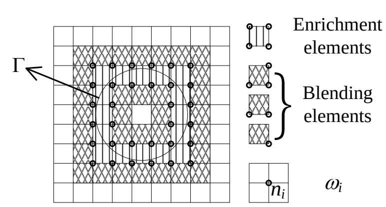
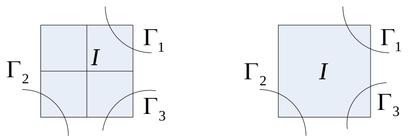


Fig. 2. Interface Γ in a non-conforming mesh.

The improved XFEM magnetic vector potential approximation can be expressed by

$$\mathbf{u}_h(\mathbf{x}) = \sum_{i \in I} N_i(\mathbf{x})\mathbf{u}_i + \sum_{k}^{N} \sum_{j \in J^{ek}} N_j(\mathbf{x})\psi_{(k)}(\mathbf{x})\left(\varphi_{(k)}(\mathbf{x})\right)\mathbf{a}_{j(k)}. \quad (1)$$

where $\mathbf{u}_i$ and $\mathbf{a}_j$ are nodal unknowns, $N_i(\mathbf{x})$ and $N_j(\mathbf{x})$ the finite element shape functions, $\psi(\mathbf{x})$ is the enrichment function, which contains the desirable discontinuous properties, and J^e the set of enrichment nodes. k = 1, 2, … , N which is the indexing of the interface Γ_k. $\mathbf{x}$ is the coordinate variable. When multiple interfaces cross the support of node I or element I as in Fig. 3 in 2D, multiple level set functions and enrichment functions have to be considered in order to preserve the convergence of the approximation.



(a) (b)

Fig. 3. Two types of mesh with nearby interfaces Γ_1, Γ_2, and Γ_3: (a) support of node I, and (b) element I.

An elegant methodology to construct the enrichment function is the level set method (LSM).

LSM is based upon the idea of representing the interface as a zero level set curve of a higher-dimensional function $\varphi(x, t)$. In this paper, only the static interfaces are considered. In the case with several interfaces $\Gamma_{(k)}$ ($k = 1, 2, \ldots , N$) as shown in Fig. 3, one function for each associated region $\Omega_{1(k)}$ can be defined as

$$\varphi_{(k)}(x) = \pm \min \left\| x - x_{\Gamma(k)} \right\|. \tag{2}$$

The sign is arbitrarily chosen negative in Ω_1 (inside Γ) and positive in Ω_2 (outside Γ).

Finally, the level set function can be obtained by

$$\varphi(x) = \min_{k=1,2,\ldots,N} \varphi_{(k)}(x). \tag{3}$$

In this work only the weak discontinuity cases will be considered because nearly all the field approximations are continuous in the electromagnetic field. The enrichment function $\psi(x)$ proposed by [7] can be used.

$$\psi(x) = \sum_i N_i(x) \cdot |\varphi_i| - \left| \sum_i N_i(x) \cdot \varphi_i \right|. \tag{4}$$

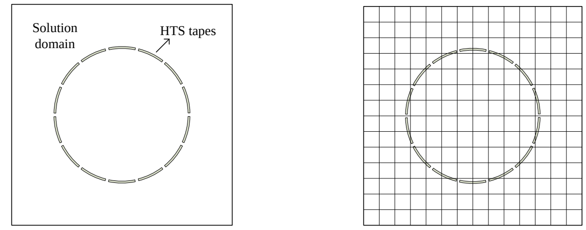
The enrichment elements must be subdivided where each sub-element contains only one material because of the numerical integration. Let φ_i and φ_j denote the nodal level set values at two vertices x_i and x_j of an element in Fig. 2. Therefore, the intersection point x_p can be calculated by

$$x_p = x_i + \xi\left(x_j - x_i\right),\ \xi = -\frac{\varphi_i}{\varphi_j - \varphi_i}. \tag{5}$$

Indeed, if only one level set function and one enrichment function are considered for node I or element I, the approximation is not rich enough to make derivative field jump along the interfaces. A node or an element has to be enriched with respect to all the interfaces crossing its support or itself. Therefore, the separate level set functions and enrichment functions for each interface have to be defined. Each interface is associated with a single level set function.

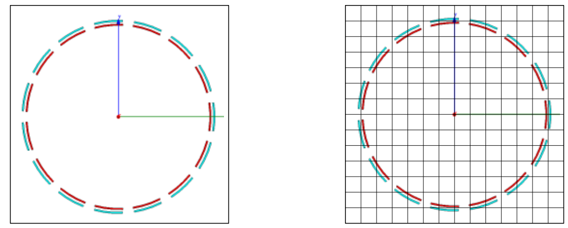
III. Numerical Examples

2D examples containing HTS tapes placed in a circular arrangement is shown in Fig. 4(a) and Fig. 5(a). The voltage loads are applied on HTS tapes, while the magnetic potentials of the outer boundaries of the air are zero. Fig. 4(b) and Fig. 5(b) show the meshes by using XFEM. Fig. 6 shows the current distribution in HTS tapes in example 2. The detail results will be presented in the full paper.



(a) (b)

Fig. 4. Example 1: (a) model, (b) XFEM mesh.



(a) (b)

Fig. 5. Example 2: (a) model, (b) XFEM mesh.

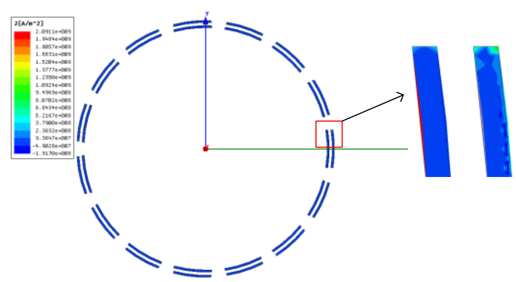


Fig. 6. Current distribution in HTS tapes in example 2.

IV. Conclusion

In this paper, a new improved XFEM is presented by using multiple enrichment functions and level sets based parts. Each interface is associated with a single level set function and additional degrees of freedom are introduced for nodes whose support is cut by more than one interface. This method is mainly used for the meshing elements containing more interfaces due to the small sizes of media.

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